$\underline{*}$ $j^k : C^\infty(M,N) \longrightarrow C^\infty(M, J^k(M,N))$, $f \mapsto j^k f$, is continuous:

Let $U \subset J^\ell(M, J^k(M,N))$ open. Then $M(U) = \{ g \mid j^\ell g(M) \subset U \}$ is open in $C^\infty(M, J^k(M,N))$ $\to$ need to show that $(j^k)^{-1}(M(U))$ is open in $C^\infty(M,N)$:

Let $\alpha_{k,\ell} : J^{k+\ell}(M,N) \to J^\ell(M, J^k(M,N))$ defined by

If $\sigma \in J^{k+\ell}(M,N)_{x,y}$ represented by $f : M \to N$, then

$$\alpha_{k,\ell}(\sigma) := j^\ell(j^k f)(x).$$

This is well-defined and smooth (Thm III.2.3).

Thus, $\alpha_{k,\ell}^{-1}(U)$ is open. Moreover, $(j^k)^{-1}(M(U)) = M(\alpha_{k,\ell}^{-1}(U))$, because for every $f \in C^\infty(M,N)$ $\alpha_{k,\ell}(j^{k+\ell} f) = j^\ell(j^k f)$ as maps $M \to J^\ell(J^k(M,N))$.

$\underline{**}$

Set $\varepsilon := \frac{1}{2} \min \left\{ d_{\mathbb{R}^n}(\operatorname{supp} \mu, \mathbb{R}^n \setminus \psi(V_i)), \; d_{\mathbb{R}^n}\left(\psi(t(\bar{S}_i)), \mu^{-1}([0,1))\right) \right\}$.

$P := \{ p \in P' \mid \| p(\varphi(x)) \|_{\mathbb{R}^n} < \varepsilon \;\; \forall x \in \operatorname{supp} \rho \}$ is then an open nbhood of 0 in P'.

We show that around (p,x) such that $F(p,x) \in \bar{S}_i$ the map $F : P \times M \to J^k(M,N)$ is a diffeomorphism (which implies $F \pitchfork S$):

$F(p,x) \in \overline{S_i} \quad \Longrightarrow \quad x \in s(\overline{S_i})$ and $G_p(x) \in t(\overline{S_i})$

Then $\delta := d_{\mathbb{R}^n}(\psi(f(x)), \psi(G_p(x))) < \varepsilon$ because

$$\psi(G_p(x)) = \rho(\varphi(x)) \cdot \mu(\psi(f(x))) \cdot p(\varphi(x)) + \psi(f(x)) .$$

Therefore

$$\delta = \| \rho(\varphi(x)) \cdot \mu(\psi(f(x))) \cdot p(\varphi(x)) \|_{\mathbb{R}^n} \begin{cases} \leq \| p(\varphi(x)) \| < \varepsilon & \text{if } \varphi(x) \in \operatorname{supp} \rho \\ = 0 & \text{if } \varphi(x) \notin \operatorname{supp} \rho . \end{cases}$$

By defn of ε and since $G_p(x) \in t(\overline{S_i})$

we have $\psi(f(x)) \in (\mu^{-1}(1))^{\circ}$. interior

Together with $\rho \equiv 1$ on a nbhd of $\varphi(s(\overline{S_i}))$, the formula for $\psi \circ G_p$ simplifies to

$$\psi(G_p(x)) = p(\varphi(x)) + \psi(f(x)) .$$

Furthermore, $G_p(x') = \psi^{-1}(p \circ \varphi + \psi \circ f)(x')$ for all x' in a nbh. of x.

Same formula holds for p' in a nbh. of p in P.

Thus, $F : P \times M \to J^k(M,N)$ is a diffeom. around (p,x): For $\sigma \in J^k(M,N)$ near $F(p,x)$ let $x' = s(\sigma)$ and p' the unique (!) polynomial of degree $\leq k$ with $\sigma = j^k(\psi(p' \circ \varphi + \psi \circ f))(x')$. The map $\sigma \mapsto (p', x')$ is smooth and inverse to F.